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On a Measure on the Inductive Limit of Projection Logics

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The aim of the paper is to measure the logic of J-projections from inductive limit of W J-algebras studied. The main result is

Theorem. *Let $\mathcal{A}$ be a W^*J-factor of countable type (type of $\mathcal{A}$ is different from I_2) and let $\mathcal{A}$ be the inductive limit of W^*J-factors $\mathcal{A}_\alpha$ different from I_2. If* (1) *$\mathcal{A}$ be a W^*P-factor or* (2) *$\mathcal{A}$ and all $\mathcal{A}_\alpha$ are W^*K-factors, then any indefinite measure $\nu : \cup_\alpha \mathcal{A}_\alpha^h \to R$ can be unique by the strong operator topology extended to an indefinite measure on A^h.*

KEY WORDS: Hilbert space; projection; measure; von Neumann algebra.

1. INTRODUCTION

Let H be a complex Hilbert space with an inner product $(\cdot, \cdot)$. Fix a self-adjoint unitary operator (=*canonical symmetry*) J (i.e., $J = J^* = J^{-1}$, $J \neq \pm I$). The form $[x, y] := (Jx, y)$ is said to be *indefinite metric* and H *indefinite metric space* (=*J-space*, =*Krein space*), see (Azizov and Iokhvidov, 1986). A vector $x \in H$ is said to be *neutral* (*positive, negative*), if $[x, x] = 0$ ($[x, x] > 0$, $[x, x] < 0$). An operator $A \in B(H)$ is called to be *J-positive* (*J-negative*) if $[Ax, x] \geq 0$ ($[Ax, x] \leq 0$) for every $x \in H$. Let $A \in B(H)$. The operator $A^\# := JA^*J$ is called to be *J-adjoint*. Note $[Ax, y] = [x, A^\# y]$, $\forall x,\ y \in H$.

Let $\mathcal{M}$ be a von Neumann algebra in H. If $J \in \mathcal{M}$ and central covers in $\mathcal{M}$ of projections $P^+ := (1/2)(I + J)$ and $P^- := (1/2)(I - J)$ are equal to I then $\mathcal{M}$ is said to be *W^*J-algebra*. If, in addition, or P^+ or P^- is finite with respect to $\mathcal{M}$ then $\mathcal{M}$ is said to be *W^*P-algebra*. A W^*J-algebra $\mathcal{M}$ is said to be W^*K-algebra

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if the W^*-algebras $P^{\pm}\mathcal{M}P^{\pm}$ contain no non-zero finite direct summand. Let $\mathcal{M}$ be a W^*J-algebra. Let us denote by $\mathcal{M}^h$, $(\mathcal{M}^{pr})$ the set of all J- self-adjoint (self-adjoint, respectively) projections from $\mathcal{M}$, i.e.,

$$\mathcal{M}^h := \{p \in \mathcal{M} : p = p^2, [px, y] = [x, py], x, y \in H\}.$$

The set $\mathcal{M}^h(\mathcal{M}^{pr})$ is said to be *hyperbolical* (*spherical*) logic. By (Azizov and Iokhvidov, 1986), $p \in \mathcal{M}^h$ is said to be *positive* (*negative*) if $[pz, pz] \geq 0([pz, pz] \leq 0)\forall z \in H$. Let $\mathcal{M}^{h+}(\mathcal{M}^{h-})$ be the set of all positive (negative) projections from $\mathcal{M}^h$. Every $p \in \mathcal{M}^h$ is representable (not uniquely!) as $p = p_+ + p_-$, where $p_\pm \in \mathcal{M}^{h\pm}$. Let us denote by e_p^+, e_p^- orthogonal projections on subspaces $\overline{P^+pH}$, $\overline{P^-pH}$ respectively.

Let $\{A_\alpha\}$ be a nondecreasing net of W^*J-algebras on H and let $\mathcal{A} := (\cup_\alpha \mathcal{A}_\alpha)''$. The algebra $\mathcal{A}$ is said to be *inductive limit* of the net $\{A_\alpha\}$. By the analogy, logics $\mathcal{A}^{pr}$ and $\mathcal{A}^h$ are said to be *inductive limits* of logics $\{\mathcal{A}_\alpha^{pr}\}$ and $\{\mathcal{A}_\alpha^h\}$. Note that if $\mathcal{A}$ is a W^*P-algebra then every $\mathcal{A}_\alpha$ is also W^*P-algebra, if $\mathcal{A}$ is a W^*K-algebra then $\mathcal{A}_\alpha$ may be W^*P-algebra.

Let $\mathcal{E}$ be one of the logics $\cup_\alpha \mathcal{A}_\alpha^{pr}$ or $\cup_\alpha \mathcal{A}_\alpha^h$. The representation $p = \sum p_i$, where $p, p_i \in \mathcal{E}$ and $p_i p_j = 0\, i \neq j$ is said to be *decomposition* of p. (The sum should be understood in the strong sense.) The function $\nu : \mathcal{E} \to R$ is said to be (*real*) *measure* if: 1) $\nu(p) = \Sigma_\beta \nu(p_\beta)$, for any decomposition $p = \Sigma_\beta p_\beta p$, $p_\beta \in \mathcal{E}$.

A measure ν is said to be *probability* if $\nu \geq 0$, $\nu(I) = 1$; *linear* measure if there exists norm continuous linear functional f on $\mathcal{A}$ such that $\nu = f/\mathcal{E}$.

Remark 1.

1. The condition (1) is essential in the classical case to continued a measure $\nu : \cup_\alpha \mathcal{A}_\alpha^{pr} \to R^+$ to a measure on $\mathcal{M}^{pr}$.
2. The property (2) $\|\nu\|(p) := \sup\{\sum |\nu(p_i)| :$ for any decomposition $p = \Sigma p_i\} < +\infty \forall p \in \cup_\alpha \mathcal{M}_\alpha^{pr}$ is equivalent one (3) M $:= \sup\{|\nu(p)|, p \in \cup_\alpha \mathcal{M}_\alpha^{pr}\} < +\infty$.

Really let (2) hold. Then $|\nu(p)| \leq \|\nu\|(p) \leq \|\nu\|(I) < +\infty$.

Now, let (3) hold. Put $p_i^+ := p_i$ if $\nu(p_i) \geq 0$, in another way $p_i^+ := 0$ and $p_i^- := p_i$ if $\nu(p_i) \leq 0$, further $p_i^- := 0$ for any decomposition $p = \Sigma p_i$. Then $\Sigma|\nu(p_i)| = \Sigma\nu(p_i^+) - \Sigma\nu(p_i^-) \leq 2M$. Hence $\|\nu\|(p) \leq 2M$.

Let $M_{\sup} := \sup\{\nu(p) : p \in \cup\mathcal{A}_\alpha^{pr}\}(\geq 0)$ and $M_{\inf} := \inf\{\nu(p) : p \in \mathcal{A}_\alpha^{pr}\}(\leq 0)$. It is easy to see that $M = \max\{|M_{\inf}|, M_{\sup}\}$, $\nu(I) = M_{\sup} + M_{\inf}$ *and* $\|\nu\|(I) = M_{\sup} - M_{\inf}$.

Note: By Dorofeev (1992) every measure ν on the set Π of all orthogonal projections in a von Neumann algebra containing no finite central summand is *bounded*, i.e. (3) $\sup\{|\nu(p)| : p \in \Pi\} < +\infty$; if dim $H < \infty$ then a measure μ is linear if and only if the property (2) holds.

The measure $\mu : \mathcal{M}^h \to R$ is said to be *indefinite measure* if $\mu/\mathcal{M}^{h+} \geq 0$ and $\mu/\mathcal{M}^{h-} \leq 0$; *semitrace* if there exists a faithful normal semifinite trace τ on $\mathcal{M}^+$ and an operator T affiliated with the center of $\mathcal{M}$ such that or $P^+T \in L_1(\mathcal{M}, \tau)$ and then $\mu(e) = \tau(Te_+), \forall e \in \mathcal{M}^h$ or $P^-T \in L_1(\mathcal{M}, \tau)$ and then $\mu(e) = \tau(Te_-), \forall e \in \mathcal{M}^h$.

2. THE MAIN RESULTS

Proposition 2. *Let H be a J-space,* $\mathcal{M} = B(H)$ *and let* $\nu(p) := Tr(Tp)$ *be a real measure on* $\mathcal{M}^h$*, where T is a trace class operator. Then T may be chosen J-self-adjoint.*

Really Tr $(Tp) = Tr(T^* p^*) = Tr(JT^*JJp^*J)$. Hence $\nu(p) = Tr(\frac{1}{2}(T + T^{\#})p)$.

Proposition 3. *Let H be a J-space and* $T = P^+TP^+ + P^+TP^- + P^-TP^+ + P^-TP^- \in B(H)$*, where* P^+TP^+, P^-TP^- *are self-adjoint. Then T is J-self-adjoint if and only if* $P^-TP^+ = -(P^+TP^-)^*$.

The proof is straightforward.

Proposition 4. *Let H be a two-dimensional (real or complex) J-space,* $\mathcal{M}$ *be the algebra of two by two matrices on H and let* $\nu(p) := \mathrm{Tr}(Tp) + c(\dim p_+)$ *be an indefinite measure on* $\mathcal{M}^h$*. Let* $T = \begin{pmatrix} a & b \\ -\bar{b} & d \end{pmatrix}$ *in the orthonormal base* $\phi_+ \in P^+H$ *and* $\phi_- \in P^-H$*. Then: (1).* $\nu(P^+) = a + c \geq 0$*; (2).* $\nu(P^-) = d \leq 0$*; (3).* $|b| \leq (1/2)(a - b)$

Proof: It is easily seen that any one-dimensional positive J-projection have the form

$$p_x := \begin{pmatrix} x & (x^2 - x)^{1/2}e^{i\theta} \\ -(x^2 - x)^{1/2}e^{-i\theta} & -(x-1) \end{pmatrix}, \quad x \geq 1, \quad \theta \in [0, 2\pi).$$

in the base ϕ_+, ϕ_-. Then

$$\nu(p_x) = \mathrm{Tr}(Tp_x) + c = \mathrm{Tr}\begin{pmatrix} a & b \\ -\bar{b} & d \end{pmatrix}\begin{pmatrix} x & (x^2 - x)^{1/2}e^{i\theta} \\ -(x^2 - x)^{1/2}e^{-i\theta} & -(x-1) \end{pmatrix} + c$$

$$= (a - d)x + d - 2(x^2 - x)^{1/2}\Re(be^{-i\theta}) + c \geq 0. \quad (1)$$

If $x = 1$ then the projection p_1 is equal to P^+. Therefore, $\nu(P^+) = \nu(p_1) = a + c \geq 0$. □

The projection $p_x^\perp := \begin{pmatrix} -(x-1) & -(x^2-x)^{1/2}e^{i\theta} \\ (x^2-x)^{1/2}e^{-i\theta} & x \end{pmatrix}$ is one-dimensional and negative. Therefore

$$\nu(p_x^\perp) = (d-a)x + a + 2(x^2-x)^{1/2}\Re(be^{-i\theta}) \leq 0. \tag{2}$$

This means that $\nu(P^-) = \nu(p_1^\perp) = d \leq 0$.

Let us divide the left-right side of (1) (or (2)) on x and let $x \to +\infty$. By the arbitrariness of $\theta \in [0, 2\pi)$, we have $|b| \leq (1/2)(a-d)$.

Note that in the base ϕ_+, ϕ_- the operator

$$e_t := \begin{pmatrix} t & (t-t^2)^{1/2}e^{i\theta} \\ (t-t^2)^{1/2}e^{-i\theta} & 1-t \end{pmatrix},$$

where $t \in [0, 1]$ and $\theta \in [0, 2\pi)$ is an one-dimensional orthogonal projection.

Proposition 5. *Let conditions of Proposition* 4 *are fulfilled and* T *from Proposition* 4. *Let us define the really measure* $\mu(\cdot)$ *on* $B(H)^{(pr)}$ *by* $\mu(e_t) := \mathrm{Tr}(TJe_t)$ *Then*

$$|\mu(e_t)| \leq 2(\nu(P^+) + |c|(\mathrm{Tr}(P^+) + |\nu(P^-)|). \tag{3}$$

Proof: By 1, 2, 3 of Proposition 4,

$$\mu(e_t) = \mathrm{Tr}(TJe_t) = \mathrm{Tr}\begin{pmatrix} a & b \\ -\bar{b} & d \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} t & (t-t^2)^{1/2}e^{i\theta} \\ (t-t^2)^{1/2}e^{-i\theta} & 1-t \end{pmatrix}$$

$$= (a+d)t - d - 2(t-t^2)^{1/2}\Re(be^{-i\theta})$$

$$\leq |a| + |d| + 2|b| \leq 2(|a| + |d|) \leq 2(\nu(P^+) + |c| + |\nu(P^-)|)$$ □

In Matvejchuk (1993) the probability measure on the inductive limit of von Neumann algebras was characterized. The following theorem is a generalization of this result.

Theorem 6. *Let a von Neumann algebra* $\mathcal{A}$ *of countable type not contained any type* I_2*-direct summand be inductive limit of von Neumann algebras* $\mathcal{A}_\alpha$ *not contained any* I_2*-direct summand. Let the measure* $\nu : \cup_\alpha \mathcal{A}_\alpha^{pr} \to R$ *be such that there exist projections* $P^+, P^- \in \cup_\alpha \mathcal{A}_\alpha^{pr}$, $P^+ + P^- = I$ *and a number* c *with the properties:*

(1) *the restriction of* ν *on* $\{p \in \cup_\alpha \mathcal{A}_\alpha^{pr} : p \leq P^+\}$ *and on* $\{p \in \cup_\alpha \mathcal{A}_\alpha^{pr} : p \leq P^-\}$ *is continued to the linear complete additive self-adjoint functional* $(:= \tilde{\nu}_\pm(\cdot))$;
(2) $|\nu(p)| \leq c(\nu(e_p^+)| + |\nu(e_p^-|)\forall p \in \cup_\alpha \mathcal{A}_\alpha^{pr}$.

Then the measure ν can be unique by the strong operator topology extended to a bounded measure on $\mathcal{A}^{pr}$.

Proof: By conditions (1), (2), $|\nu(p)| \leq c(\|\tilde{\nu}_+\| + \|\tilde{\nu}_-\|)$ for all $p \in \cup_\alpha \mathcal{A}_\alpha^{pr}$. Hence $\|\nu(I)\| < +\infty$. □

Let ϕ be a faithful normal state on $\mathcal{A}$. Let $p \in \mathcal{A}^{pr}$ and let a sequence $\{g_n\} \subset \cup_\alpha \mathcal{A}_\alpha^{pr}$ convergent to p in the strong operator topology ($g_n \xrightarrow{s} p$ for brevity). Put $\Delta_{n,m} := \phi(g_n g_m^\perp g_n)$. By the construction of Gunson (1972), there exist decompositions $g_n = g_n^0 + g_n^1$, $g_m = g_m^0 + g_m^1$, $g_n^0, g_n^1, g_m^0, g_m^1 \in \cup_\alpha \mathcal{A}_\alpha^{pr}$ such that

$$\phi\big(g_n^1\big) \leq \Delta_{n,m}, \quad \phi\big(g_m^1\big) \leq \phi^{1/2}((g_n - g_m)^2) + \Delta_{n,m} + \Delta_{n,m}^{1/2}$$

and $\|g_n^0 - g_m^0\| \leq \Delta_{n,m}^{1/2}$.

By Matvejchuk (1991) Lemma 7 (see also Gunson, 1972, Theorem 2.11),

$$|\nu(e) - \nu(f)| \leq 9\|\nu\|(I)\|e - f\|^{1/2}, \quad e, f \in \mathcal{A}_\alpha^{pr}, \quad \forall\alpha.$$

By (2) of Theorem 6,

$$\begin{aligned} |\nu(g_n) - \nu(g_m)| &\leq \big|\nu\big(g_n^0\big) - \nu\big(g_m^0\big)\big| + \big|\nu\big(g_n^1\big)\big| + \nu\big(g_m^1\big)\big| \\ &\leq 9\|\nu\|(I)\big\|g_n^0 - g_m^0\big\|^{1/2} + \big|\nu\big(g_n^1\big)\big| + \big|\nu\big(g_m^1\big)\big| \\ &\leq 9\Delta_{n,m}^{1/4} + c(|\nu(e_{g_n^1}^+)| + |\nu(e_{g_n^1}^-)|) + c(|\nu(e_{g_m^1}^+)| + |\nu(e_{g_m^1}^-)|). \end{aligned}$$

In the paper Matvejchuk (1982) see the proof of Lemma (2) it was shown that

$$f \xrightarrow{s} 0 \quad \text{implies} \quad e_f^+ \xrightarrow{s} 0 \quad \text{and} \quad e_f^- \xrightarrow{s} 0. \tag{4}$$

Hence $e_{g_n^1}^+ \xrightarrow{s} 0$, $e_{g_n^1}^- \xrightarrow{s} 0$, $e_{g_m^1}^+ \xrightarrow{s} 0$, $e_{g_m^1}^- \xrightarrow{s} 0$. Therefore by (1) of Theorem 6,

$$|\nu(e_{g_n^1}^+)| + |\nu(e_{g_n^1}^-)| + |\nu(e_{g_m^1}^+)| + |\nu(e_{g_m^1}^-)| \to 0 \quad n, m \to \infty.$$

This means that the sequence $\{\nu(g_n)\}$ is fundamental. Put $\tilde{\nu}(p) := \lim \nu(g_n)$. It is clear that $\tilde{\nu}(p)$ is well defined.

(i) Note now if $g_n \xrightarrow{s} p$, $\{g_n\} \subset \cup_\alpha \mathcal{A}_\alpha^{pr}$ then $\{e_{g_n}^+\} \xrightarrow{s} e_p^+$ and $\{e_{g_n}^-\} \xrightarrow{s} e_p^-$. Hence by the definition of $\tilde{\nu}(\cdot)$ and by (1) and (2) of Theorem, 6 we have:

$$|\tilde{\nu}(p)| \leq c(\tilde{\nu}(e_p^+) + \tilde{\nu}(e_p^-)),\ p \in \mathcal{A}^{pr} \tag{5}$$

Now let $p_n \subset \mathcal{A}^{pr}$ and $p_n \xrightarrow{s} 0$. Then by the complete additivity of $\tilde{\nu}(\cdot)$ on $\{e \in \mathcal{A}^{pr} : e \leq P^\pm\}$ (see the condition (1) of Theorem 6) and by (5),

(a) $\{p_n\} \subset \mathcal{A}^{pr}$ and $p_n \xrightarrow{s} 0$ implies $\tilde{\nu}(p_n) \to 0$.

(ii) Let $e_1, e_2 \in \mathcal{A}^{pr}$, $e_1 \perp e_2$, $\{g_m\}_1^\infty \subset \cup_\alpha \mathcal{A}_\alpha^{pr}$ and $g_m \xrightarrow{s} e = e_1 + e_2$ then there exist the sequences $\{g'_m\}$ and $\{g''_m\} \subset \cup_\alpha \mathcal{A}_\alpha^{pr}$ such that $g_m = g'_m + g''_m$ and $g'_m \xrightarrow{s} e_1$, $g''_m \xrightarrow{s} e_2$. Therefore $\tilde{\mu}(\cdot)$ is additive on $\mathcal{A}^{pr}$

$$\tilde{\nu}(e_1 + e_2) = \tilde{\nu}(e_1) = \tilde{\nu}(e_2).$$

By (a), $\tilde{\nu}(\cdot)$ is complete additive. Thus for $\tilde{\nu}(\cdot)$ the condition (1) of the definition of the measure is fulfilled.

By (3), $|\tilde{\nu}(p)| \leq c(\|\tilde{\nu}_+\| + \|\tilde{\nu}_-\|)$.

In Matvejchuk (2000) Theorem 4, we examined indefinite measures on W^*J-algebra (the case B(H) see also Matvejchuk (1991). We have proved:

Let $\mathcal{P}$ be the logic of all J-self-adjoint projections from a W*J-algebra $\mathcal{B}$ acting in a space with an indefinite metric containing no central summand of type $I_{n,m}(n, m \leq 2)$. Then for every indefinite measure $\nu : \mathcal{B}^h \to R$ there is J-self-adjoint trace-class operator T such that:

(i) If $\mathcal{B}$ is a W* P-algebra, then

$$\nu(p) = \mathrm{Tr}(Tp) + \nu_0(p), \quad \forall p \tag{6}$$

for some semitrace ν_0

(ii) If $\mathcal{B}$ is a W^*K-algebra, then $\nu(p) = \mathrm{Tr}(Tp), \quad \forall p.$

The main result of the paper is

Theorem 7. *Let $\mathcal{A}$ be a W^*J-factor of countable type (type of $\mathcal{A}$ is different from I_2) and let $\mathcal{A}$ be inductive limit of W^*J-factors $\mathcal{A}_\alpha$ different from I_2. If*

(1) *$\mathcal{A}$ be a W^*P-factor or*
(2) *$\mathcal{A}$ and all $\mathcal{A}_\alpha$ are W^*K-factors, then any indefinite measure $\nu : \cup_\alpha \mathcal{A}_\alpha^h \to R$ can be unique by the strong operator topology extended to an indefinite measure on $\mathcal{A}^h$.*

Proof: Let us establish, for instance, the case (1). (For proof of the case (2) we can proceed analogously). Any central operator from $\mathcal{A}$ is equal to tI. Let τ be a faithful normal semifinite trace on $\mathcal{A}^+$. Without loss of generality we may assume that $VP^+V^* \leq P^-$ for some partial isometry $V \in \mathcal{A}$ and $\tau(P^+) < +\infty$. Thus by (6), there exists a weakly continuous on the unit sphere of $\mathcal{A}_\alpha$ J-self-adjoint linear functional f_α such that $\nu(p) = f_\alpha(p) + t_\alpha \tau(p_+)$, $\forall p \in \mathcal{A}_\alpha^h$.

By uniqueness of semitrace $t_\alpha \tau(p_+)$ on $\mathcal{A}_\alpha^h$ and by nondecreasing of $\{\mathcal{A}_\alpha\}$, we conclude that the number t_α does not depend on α. Thus $t_\alpha = t$ for all α.

By uniqueness of $f_\alpha(\cdot)$ we conclude that $\alpha \leq \beta$ implies $f_\alpha(p) = f_\beta(p)$ for all $p \in \mathcal{A}_\alpha$. The functional $f_\alpha^J(\cdot) := f_\alpha(J\cdot)$ on $\mathcal{A}_\alpha$ is self-adjoint. Put $\mu(p) := f_\alpha^J(p)$ if $p \in \mathcal{A}_\alpha^{pr}$ for all α.

Let us show that $\mu : \cup_\alpha \mathcal{A}_\alpha^{pr} \to R$ is a bounded measure. It is clear that μ is a finite additive function. It is sufficient to proof that μ is strong continuous at 0. The restriction of μ onto $\{p \in \cup_\alpha \mathcal{A}_\alpha^{pr} : p \leq P^+\}(\{p \in \cup_\alpha \mathcal{A}_\alpha^{pr} : p \leq P^-\})$ is a measure positive (negative, respectively). By Theorem of Matvejchuk (1993), this restrictions are strong continuous at 0.

Let $p \in \cup_\alpha \mathcal{A}_\alpha^{pr}$. The minimal weakly closed *-algebra of operators $(:= \mathcal{A}(p))$ generated by orthoprojections $p, e_p^+ e_p^-$ is the direct integral of factors of type I_2. The restriction of ν on J-projections from $\mathcal{A}(p)$ is an indefinite measure. Let us apply Proposition 5 to the restriction. This means that the inequality

$$|\mu(p)| \leq 2(\nu(e_p^+) + |t|\tau(e_p^+) + |\nu(e_p^-)|). \tag{7}$$

is true. By (4) and (7), $\mu(p) \to 0$. Hence μ is a measure. By (7) again, $|\mu(p)| \leq 2(\|\nu_h\| + |t|\tau(P^+) + \|\nu_h\|)$. Therefore $\mu(\cdot)$ is bounded.

Thus for μ conditions of theorem 6 are fulfilled. By Theorem (Matvejchuk, 1995), there exists a weakly continuous on the unite sphere of $\mathcal{A}$ selfadjoint linear functional $(:= g(\cdot))$ such that $\mu(\cdot) = g(\cdot)/\mathcal{A}_\alpha$. It is clear that the formula $\bar{\nu}(p) := g(Jp) + t\tau(p_+)$, $p \in \mathcal{A}^h$ in the case 1) and $\bar{\nu}(p) := g(Jp)$, $p \in \mathcal{A}^h$ in the case 2) give us the continuation.

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